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This Journal is dedicated to the following aims:

1. THROUGH PUBLISHED STANDARD PAPERS ON THE CULTURE ASPECTS, HUMANISM AND HISTORY OF MATHEMATICS TO DEEPEN AND TO WIDEN PUBLIC INTEREST IN ITS VALUES.
2. TO SUPPLY AN ADDITIONAL MEDIUM FOR THE PUBLICATION OF EXPOSITORY MATHEMATICAL ARTICLES.
3. TO PROMOTE MORE SCIENTIFIC METHODS OF TEACHING MATHEMATICS.
4. TO PUBLISH AND TO DISTRIBUTE TO THE GROUPS MOST INTERESTED HIGH-CLASS PAPERS OF RESEARCH QUALITY REPRESENTING ALL MATHEMATICAL FIELDS.

The University of Wisconsin and Mathematics

Gratifying to all upholders of the supreme value of mathematics as a foundation study will be the following announcement, sent to us by Professor W. W. Hart. We quote a portion of the document forwarded, namely, "Recommendations of the Committee on Entrance Requirements," said document having been "approved by the University (of Wisconsin) Faculty at its adjourned meeting of June 6, 1934:"

" 'Unrestricted' admission to the University is admission which opens to the student all Colleges, Courses, and fields of study to which freshmen are eligible and insures full freedom of choice among all the college majors and fields of specialization. 'Restricted' admission opens to the student such Colleges, Courses, and fields of specialization as do not require high school mathematics as background. It does not give admission to the College of Agriculture or the College of Engineering or the Course in Chemistry and does not permit the student to major or specialize in chemistry, commerce, economics, mathematics, pharmacy, pre-medicine, philosophy, political science, psychology or sociology, or in any of the other natural sciences including physical geography, and geology, or to graduate from the School of Education with a major or minor in any of these fields. . . . . 'Restricted' standing may at any time at the option of the student be changed to 'unrestricted' standing by the mastery, subsequent to high school graduation, of the content of high school algebra and plane geometry by private study, tutoring, or correspondence study. The University will not provide instruction in this preparatory work."

Mathematics in the Liberal Arts College

HARVEY A. WRIGHT
Transylvania College

"The teaching of the college is grounded upon the principle that the right use of physical and intellectual powers is determined by the dominance of spiritual values." "From the standpoint of culture the college seeks to give the student an understanding of the material and social world in which he lives and does his work. It seeks to give him an appreciation of man's achievements in civilization and the part that literature, science, history, institutions and customs have played in this process." "In keeping with these principles, the college believes that the cultural aspects of education should be united with such useful knowledge and skill as will enable the student to live effectively and to contribute his full share to the productive work of the world." These quotations are from 'The Objectives of the College' as found in the current catalogue (page 25) of Transylvania College and are assumed in this paper to be an acceptable statement of objectives of the Liberal Arts College.

The only phrase, in the list of objectives above, which might suggest mathematics to many persons is, 'useful knowledge and skill that will enable the student to live,' for in the minds of a large number of people, mathematics is associated with the so called practical things of life and by practical they mean something that can be turned into money or its equivalent. It is almost impossible for such a person to realize that mathematics has a place in the college curriculum other than for those students who expect to enter the engineering profession or for students who naturally like to study the subject. Can mathematics assist the college in the development of spiritual values in its students, that will 'dominate the right use of physical and intellectual powers?' Is it possible for mathematics to aid the student in his attempt to 'understand the material and social world in which he lives and does his work?' Can the mathematics department help integrate the cultural aspect of education and useful knowledge and skill so as to enable the student 'to live effectively and to contribute his full share to the productive work of the world?' These questions will not be answered categorically, if at all, but they are in the mind of the author as this paper is being written.

Almost every speaker who addresses a convocation of college students refers to changes that have taken place since he was in college. It is true of course that changes have occurred but it is also true that

much of the world in which the student lives and does his work is not changing. There are many constants in the world of the student and these unchanging factors must be known and understood before he can appreciate his world. The planet upon which the student lives is three-fourths water and one-fourth land and is surrounded by an atmosphere. What is the student to do about it? There is nothing he can do about it except to discover and understand the laws which apply to these elements and then adjust his life accordingly. The earth has been in motion ever since there was an earth but man did not always know this fact. The discovery of planetary laws and the realization that they are unchanging has profoundly affected the philosophy and deeds of mankind. Much superstition dissappeared and constructive thinking took its place. Man's achievements in civilization are the results of the discovery of that which changeth not, and it is hardly necessary to add that mathematics, directly and indirectly, has had a part in the discovery and formulation of these laws.

Mathematics not only aids in the discovery of eternal laws but the laws of mathematics are themselves permanent. The Pythagorean Theorem is as true today as when it was first formulated and no one need hesitate to use it when making plans for today or the far distant future. Permanence instills confidence. In a world of social, political and economic turmoil and uncertainty it is comforting to know that the universe itself is running smoothly in accordance with eternal laws, and that the principles by which these laws were discovered are still available to those who will understand and use them.

The supervisor of mathematics in a city school system visited a sixth grade arithmetic class. The teacher wrote the same number (minuend) several times in a horizontal line on the blackboard, and under each minuend she wrote a subtrahend, no two subtrahends being equal. She then asked the class to indicate without actually performing the subtractions, where the greatest difference would occur. Functionality, yes, but of course not by that name. Those sixth graders were learning that when the minuend remains constant, a change in the subtrahend produces a change in the difference, and furthermore they learned that as the subtrahend increased the difference decreased, and vice versa. Those pupils are now adults and have met many life problems. Whether or not they attempt to solve their life problems by seeking to determine which factors are constant or what effect a change in one may have on another, the writer does not know, but he is confident that many life situations can be attacked by the functional method. The variety of functional relationships are somewhat limited in the sixth grade but the student who has continued his study of mathematics through college

has had an opportunity to build a broad foundation for functional thinking.

Students frequently ask, "How many decimal places do you want us to carry?" This and other questions involving approximations affords an opportunity to develop a mode of thinking which finds application in situations not strictly mathematical. A tenth of an inch in some situations is negligible while in others it is very significant. In like manner, factor k may be of little importance in the analysis of some life situations and its effect approximated, while in others it may need the most careful consideration. Mathematics is often spoken of as an exact science, and as such it affords an opportunity to develop the habit of accuracy and of precise thinking. It might seem that approximations would tend to develop loose and careless thinking but that is not necessarily true. On the other hand the student must think clearly and comprehendingly if he is to make intelligent and satisfactory approximations. Then too, approximations may be used as a check when accurate conclusions are desired. A base ball umpire does not have much time to check the accuracy of his decisions but most of life situations allow ample time.

Considerable discussion was provoked recently by a question which a young man asked the editor of "The American Magazine," namely, "Does it pay to be honest?" Apparently the young man really had in mind the question of success and honesty. A mathematical approach to this question is that of first determining, if possible, the necessary and sufficient conditions for success then considering the effect of honesty, also dishonesty, upon these conditions. This method is of course applicable to a variety of situations.

A few days ago a college senior told me that he had had but one year of mathematics, "but," he added, "I am glad I had that much because it helped me to understand generalization." So we add another mathematical process to our list of processes which are applicable to life situations.

Mathematics would doubtless be a source of delight and an inspiration to many more students if we knew better how to teach the subject. Many college freshmen remember their high school algebra well enough to "transpose" when solving certain equations but most of them do not know what "transposing" actually is. Some freshmen, when they enter college, solve linear equations having fractional coefficients by first writing the terms over a common denominator and then "dropping" the denominator. These pupils seldom (never in the writer's experience) know what they are actually doing. The lack of comprehension is by no means limited to high school graduates. A college graduate who is teaching physics in a secondary school once asked

the writer to recommend a calculus text that would tell him, "what it was all about." He had studied calculus in college, received a mark of B, but never found out what calculus really is. Such a person could not possibly be enthusiastic about mathematics. Our analytical geometries give four type equations for the straight line. These equations, to most students, are just so many formulas to be learned and the various sets of conditions are signals to perform certain sets of skills. The student almost never understands the linear function from a reading of the text. The text book and the teacher are not to be blamed for all of the lack of understanding; the perfect text and the perfect teacher are non-existent but that does not disprove the statement that understanding is a necessary condition to full appreciation. Many pupils, perhaps some teachers also, 'like' mathematics much as they like to work puzzles, they enjoy this particular form of physical activity. They fail to enjoy mathematics to the full, however, because they fail to comprehend its real essence.

When a student in college, the writer listened to a series of talks on such topics as "A Geologist's Religion," "Why a Physicist Believes in God," and similar talks by a botanist and a chemist. Each speaker humbly told of the debt he owed to his field of study because through it he had learned to know his maker better and consequently learned better how to worship. There are those who feel toward mathematics as each of these men felt toward his science. Professor Carmichael once said, "Mathematical thought has exerted over my spirit a fascination which is far-reaching in its effect." Not long ago an auto mechanic who was standing near an automobile whose motor was running, turned and said, "Don't she purr purty." Now anyone would rather listen to the gentle purr of a new modern motor than to the rattle of a worn out machine, but it was not the purr that really held the attention of the mechanic. He was visualizing what was going on under the hood; perfect coordination of moving parts, some large some small, each doing its duty at the proper time. One wonders if a mechanic does not regard a machine as having a soul. Be that as it may, there is more in mathematics than formula and skills, no matter how valuable the formulas or how fascinating the skills. This 'more abundant life' of mathematics may be experienced by some students in spite of the teacher and the text but determined effort and enthusiasm on the part of the instructor should result in more 'conversions.' Most students will not realize the value of mathematical thought processes unless their attention is directed to such values, therefore the professor should do more than teach, he should preach mathematics in class and out of class. If the student him-

self becomes a teacher he will 'carry the gospel' to his own pupils, but even if he does not teach he will spread the 'good news.'

This paper is merely suggestive: dream your own dream of mathematics, assisting your college in its attempt to realize its objectives, cultural as well as utilitarian, then awake and to work.

To Construct a Magic Square of Order 2n From a Given Square of Order n

By A. L. CANDY
University of Nebraska

[Editor's note: This is the second and final installment of a paper, the first part of which was published in the April, 1934 issue of the Mathematics News Letter. In that part the author, using the method of Current Groups, describes and illustrates a process by which from a symmetric magic square of even order n may be generated many symmetric squares of order $2n$. If the initial square is of odd order n and symmetric, squares of order $2n$ may be generated that are symmetric except with respect to two pairs of numbers.

In the following paper he shows that the groups of four numbers defining a cell of the square need not be consecutive.

Due to space limitations the squares to which foot-note references are made are not here reproduced since it is assumed that our readers have access to the original issue in which they appeared. No. 7, Volume 8 of the Mathematics News Letter.]

In our former discussions it has been assumed that the groups have each consisted of four consecutive numbers. This is not necessary. The first group may be the first four terms of any arithemetic series, the the second group, the second four terms, etc. But this would extend the series beyond the first $4\ n^2$ consecutive integers. But these groups may be chosen so that the numbers in the different groups shall have the same common difference, and the total number in the series be $4\ n^2$. Furthermore, all that has been said about the order of writing the groups in the 2 x 2 squares will still hold.

For example let $n^2=ds$, where d and s are both integers. Then, since $4n^2=d(4s)$ let the series of $4n^2$ consecutive numbers be written in the form of a rectangular array in which the columns are written in regular numerical order with d numbers in each column, and $4s$ numbers in each row. We thus get a set of arithmetic series having the same common difference d, with $4s$ terms, or s groups, in each series. Since these series begin respectively with the first d consecutive numbers, they may be called Consecutive Arithmetic Series. Now the groups must be taken consecutively from these consecutive arithmetic series. That is, the first s cells of the given square must be filled with the s groups of the

first series, the second s cells with the s groups of the second series, etc. A part of a group, when chosen in this way, cannot be taken from one series and the remainder from the next series.

As the simplest case we may take $d=2$. Then $n^2=2(\frac{1}{2}n^2)$ and $4s=2n^2$ which is divisible by four only when n is even. The two series consist of the odd and even numbers respectively. The first half of the cells will be filled with consecutive groups of odd numbers, and the second half with consecutive groups of even numbers. Fig. 14[4] was formed this way. But in any case when $d=2$ the groups may be taken alternately from the two series. That is, odd numbered cells may be filled with the groups of odd numbers and even numbered cells with the groups of even numbers, as done in Figs. 15[1] and 18. (In square No. 15[4] all groups are written in crossed order.)

When n is odd these consecutive series can be formed in two ways. (1) $n^2=n\times n$. This gives n series with n groups, or $4n$ terms in each series, with common difference n. (2) Since we may write $n^2=n^2\times 1$, we may form n^2 series with 1 group, or 4 terms, in each series, with common difference n^2.

Therefore, when $n=3$, for example, we have the following rectangular arrays from which we can select the groups to form a 6×6 square from the 3×3 square.

(1) $d=n=3$, gives

1	4	7	10		13	16	19	22		25	28	31	34
2	5	8	11		14	17	20	23		26	29	32	35
3	6	9	12		15	18	21	24		27	30	33	36

with three groups in each row, or series. See Fig. 2.[2]

(2) $d=n^2=9$ gives

1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18
19	20	21	22	23	24	25	26	27
28	29	30	31	32	33	34	35	36

When thus written the columns form the groups. See Fig. 3.[2]

For the corresponding cases when $n=5$, see Figs. 8,9.[2] Observe that Fig. 9 is made from a different 5×5 square.

When n is even d may have several other values besides n, and n^2. For example in Fig. 16,[4] $d=2n=8$, and there are two groups in each series.

When d is 4 or some multiple of 4 these concurrent groups may be taken consecutively from the first groups in each of the arithmetic series, then from the second in each series, etc. Fig. 20, in which $n=6$, was constructed in this manner. Since $n=6$, $n^2=ds$ was taken in the form

17	21	31	27	143	139	136	132	59	63	53	49
	5		7		35		36		15		13
29	25	19	23	131	135	140	144	55	51	57	61
20	24	30	26	142	138	133	129	58	62	56	52
	8		6		34		33		14		16
32	28	18	22	130	134	137	141	54	50	60	64
97	101	111	107	79	75	72	68	43	47	37	33
	25		27		19		20		11		9
109	105	99	103	67	71	76	80	39	35	41	45
100	104	110	106	65	69	74	78	42	46	40	36
	28		26		17		18		10		12
112	108	98	102	77	73	70	66	38	34	44	48
81	85	95	91	4	8	11	15	123	127	117	113
	21		23		4		3		31		29
93	89	83	87	16	12	7	3	119	115	121	125
84	88	94	90	2	6	9	13	122	126	120	116
	24		22		2		1		30		32
96	92	82	86	14	10	5	1	118	114	124	128

Fig. 20. $d=4$ $n^2=ds=36=4\cdot 9$ First four squares are filled with first groups of each series.

$36=4\times 9$. This gives 4 series with 9 groups in each series. The first 4 cells of the 6×6 square were filled with the first group in each of these four series. The groups might also have been taken in this order in Fig. 16[4] where $d=2n=8$, and in Fig. 19 where $d=\frac{1}{2}n^2=32$.

The groups considered thus far have each been composed of four consecutive terms of the same arithmetic series. This, however, is not necessary. The first and second numbers may be taken from one series of consecutive integers and the third and fourth numbers must then be the corresponding terms of another such series.

Let a_1 and a_2 be the two numbers taken from the first series

Let a_3 and a_4 be the two numbers taken from the second series

Let $d_1=a_2-a_1=a_4-a_3$ and $d_2=a_3-a_1=a_4-a_2$

1	33	128	96	159	191	226	194	67	99	62	30	221	253	164	132
	1		63		62		4		5		59		58		8
97	65	32	64	255	223	130	162	35	3	94	126	189	157	196	228
156	188	229	197	6	38	123	91	218	250	167	135	72	104	57	25
	56		10		11		53		52		14		15		49
252	220	133	165	102	70	27	59	186	154	199	231	40	8	89	121
152	184	233	201	10	42	119	87	214	246	171	139	76	108	53	21
	48		18		19		45		44		22		23		41
248	216	137	169	106	74	23	55	182	150	203	235	44	12	85	117
13	45	116	84	147	179	238	206	79	111	50	18	209	241	176	144
	25		39		38		28		29		35		34		32
109	77	20	52	243	211	142	174	47	15	82	114	177	145	208	240
17	49	112	80	143	175	242	210	83	115	46	14	205	237	180	148
	33		31		30		36		37		27		26		40
113	81	16	48	239	207	146	178	51	19	78	110	173	141	212	244
140	172	245	213	22	54	107	75	202	234	183	151	88	120	41	9
	24		42		43		21		20		46		47		17
236	204	149	181	118	86	11	43	170	138	215	247	56	24	73	105
136	168	249	217	26	58	103	71	198	230	187	155	92	124	37	5
	16		50		51		13		12		54		55		9
232	200	153	185	122	90	7	39	166	134	219	251	60	28	69	101
29	61	100	68	131	163	254	222	95	127	34	2	193	225	192	160
	57		7		6		60		61		3		2		64
125	93	4	36	227	195	158	190	63	31	66	98	161	129	224	256

FIG. 19 $d=\frac{1}{2}n^2=32$ 16x16

Such groups may be selected in the following three ways when n is either odd or even. This method can be made sufficiently clear by illustrating it with the simplest case when $n=3$.

First, $d_1=1$ and $d_2=2n^2=18$. Write the first half of the series of numbers ($18=2n^2$) in numerical order, and write the second half im-

mediately below, also in numerical order. Draw vertical lines between each alternate pair of these numbers thus:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36

The successive groups of numbers between these lines give the necessary $9(n^2)$ consecutive groups. The groups in Fig. 4^2 where $n=3$, Fig. 10^6 where $n=5$, were selected in this manner.

Second, $d_1=1$ and $d_2=2n=6$. Write the series of $36(\text{or}4n^2)$ numbers in the form of a square, the rows being written in numerical order. Draw lines between alternate rows and columns thus forming small squares with 4 numbers in each square. These groups of 4 numbers taken consecutively, either as to rows or as to columns, will give the necessary 9 (or n^2) consecutive groups. The groups in Fig. 5^2 where $n=3$, and Fig. 11^5 where $n=5$, were formed in this manner.

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36

Third, $d_1=n=3$ and $d_2=2n^2=18$. Divide the same square into four 3×3(or $n\times n$) squares. In this case the groups are formed by taking one number from each of these small squares, the four numbers being similarly located. That is, for example, corners of 4×4 squares.

1	2	3	4	5	6
7	8	9	10	11	12
13	4	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36

Beginning with the upper left hand corners we get the following 9 (or n^2) groups:

First row, first column	1	4	19	22
First row, second column	2	5	20	23
First row, third column	3	6	21	24
Second row, first column	7	10	25	28
Second row, second column	8	11	26	29
Second row, third column	9	12	27	30
Third row, first column	13	16	31	34
Third row, second column	14	17	32	35
Third row, third column	15	18	33	36

See figures 6 and 12.[10]

When n is even we may take $d_1=1$ and $d_2=4n$. Or we may take $d_1=1$ and $d_2=n^2$.

When n is evenly even these groups may be selected in still other ways. In this third method, for example, each quadrant of the natural square may be divided into four parts and the process applied to each quadrant.

Groups selected by either of the above ways may be written in the same orders as previously explained when n is even. When $n=3$, a few changes are necessary. These can readily be found by comparing Figs. 3[2] and 4. When n is an odd number greater than 3, the groups in the 9 center cells may be written the same as when $n=3$, and the groups in the surrounding cells of the given square may be written almost as previously explained. The few necessary adjustments can be made without any difficulty.

When n is odd the symmetry is not quite so good. The ends of the two center rows, as well as the ends of the two columns are not symmetric with respect to the center.

It is worthy of note that in any of these squares the sums of these 2×2 squares form a magic square of order n. This is illustrated by the Figs. 22, 23[7] and 24 where the sums of the 2×2 squares in the 12×12 square of Fig. 22 form the 6×6 square of Fig. 23. In like manner this 6×6 square gives the 3×3 square of Fig. 24. This is due to the fact that these groups have been chosen so that if the first terms of these groups were written alone in the cells of the given square, the result would be a magic square of order n. Likewise with each of the other terms.

When n is evenly even groups can be selected so that the sums of these 2×2 squares shall all be the same, viz. twice the sum of the first and last terms of the series. To do this write the series of $4n$ numbers in four rows of n consecutive numbers in each row. Write the first and

third rows from left to right, and the second and fourth from right to left. Then the consecutive columns will give the required groups. Fig. 17[8] was construceted in this way.

It is possible also to construct a 6×6 square so that the sums of the 2×2 squares shall be the same, but it does not seem to be possible to select the necessary groups in any consecutive order.

As has already been said a great many different squares may be constructed by this method. For example in any of these squares two whole rows which lie in the same row of the given square—or any number of such rows—may be interchanged provided the symmetric pairs of rows also are interchanged; likewise the groups in any pair of symmetric cells of the given square, or any number of such pairs may be reversed; also, if $n > 5$, two groups which are in the same row of the given square, and balanced as to rows, may be inverted, provided the symmetric groups are also inverted, and the square will remain magic, symmetry will not be affected, and each group will remain in the same cell of the given square.

In this way at least 128 different 6×6 squares may be obtained from any of the squares in Figs. 1, 2, 3, 4, 5,[2] and 6. Similarly the number of slightly different 10×10 squares that can be obtained from Fig. 7,[10] say, is $2^{15}=32762$. If we do not care to preserve symmetry, or prefer to write more groups in the z orders, or make a more varied use of the cyclic and crossed orders, these numbers may be largely increased. When we bear in mind the large number of possible 5×5 squares, we get some idea of the tremendous number of possible 10×10 squares.

1 See pp. 156, 157 of Mathematics News Letter, No. 7, Volume 8.
2 See p. 148 of Mathematics News Letter, No. 7, Volume 8.
3 See pp. 150, 151, Mathematics News Letter, No. 7, Volume 8.
4 See p. 156, Mathematics News Letter, No. 7, Volume 8.
5 See p. 153, Mathematics News Letter, No. 7, Volume 8.
6 See p. 152, Mathematics News Letter, No. 7, Volume 8.
5 Se p. 153, Mathematics News Letter, No. 7, Volume 8.
6 See pp. 148, 152, Mathematics News Letter, No. 7, Volume 8.
7 See pp. 158, 159, Mathematics News Letter, No. 7, Volume 8.
8 See p. 156, Mathematics News Letter, No. 7, Volume 8.
9 See p. 149, Mathematics News Letter, No. 7, Volume 8.
10 See p. 154, Mathematics News Letter, No. 7, Volume 8.

The proof of the transcendency of π will hardly diminish the number of circle-squarers, however; for this class of people has always shown an absolute distrust of mathematicians and a contempt for mathematics that cannot be overcome by any amount of demonstration. —Felix Klein in the "Evanston Calloquium."

The Teacher's Department

Edited by

JOSEPH SEIDLIN AND W. PAUL WEBBER

INDIVIDUAL STUDY OF MATHEMATICS

A professor of education in a series of addresses to college teachers a few years ago arrived at the conclusion that the chief value of the teacher in the class room is to lead the pupils in the cultivation of the power and habit of group thinking. This was something of a shock to many of us who had been of the opinion that the teacher is rather important in several ways as an educational agency. Well, maybe group thinking is of major importance in the educational process and we have not recognized the fact. If one were to notice the election returns from time to time over the country one might easily conclude that group thinking has either been neglected in education or else it does not transfer to affairs of state. It has been my observation that a good deal of what is called group teaching or group thinking in the class room is little more than group listening either to the teacher or to the brighter members of the class. If there was any thinking it was concerning how to absorb enough stock in trade questions with their answers to enable the pupils to probably pass the next test with at least a D or a C. Even this thinking may have been tinctured with the flavor of individualism. In some subjects a measure of success can be attained by the "group thinking game." In technical sciences and in mathematics the game has never been a marked success. Neither does it work so well in voice culture or in football. Those who have read the recent article in a popular journal on "What you don't see in football" will recall that it was individual thinking and initiative that won many a close game. It is just in the nature of things that in many of the fundamental cultures of life there is no satisfactory substitute for individual thinking and hard individual practice or drill. The same is true in the practice of a vocation.

The teacher of Junior engineering mechanics complains that his pupils avoid in a written test all problems that require calculus to solve. Calculus is taught in the sophomore year. Again the calculus teachers daily see their pupils "blow up" when a principle of algebra or of trigonometry is required, in the study of calculus, and so on down the line. To most pupils it never occurs that they should go to their old books on algebra and trigonometry and look up what they have

forgotten or never knew. This, to them, looks too much like work. It would seem that with all the years they have spent in school most of them have not learned to study and that serious application is to be expected. They cannot (or will not) use their books with any degree of efficiency. Some do not want to study if they can guess the teacher for a D or a C and go on to the next year's work. It appears that one of the first duties of the mathematics teacher in college is to teach the need of, and how to study from, books. This is an individual matter and is not so simple either. For there are many pleasant distractions on most campuses.

One thing administrative agencies can do is to encourage to the fullest extent the holding of students to a reasonably high standard. In mathematics the time allotment for courses is frequently too short for the course to be seriously covered. Here administrative agencies can aid by authorizing desirable changes in schedules. Again it occurs often in rapidly growing institutions that the class room equipment is sadly deficient for good teaching. In a subject like mathematics facilities for much individual drill and practice are necessary.

One teacher proposes to teach college mathematics by laboratory methods much as engineering drawing is taught to first year students. For example, assign to a teacher and a student assistant a fairly large class in a room equipped for study and practice, i.e. with tables or arm chairs. Let each pupil work at his own pace until he actually learns the subject. When he is through give the examination and pass him on to the next course. Give lectures or general class exercises only occasionally when it seems to be the best way to accomplish a specific thing. The lectures would naturally be of the nature of giving the orientation and purpose of the different topics or chapters of the course.

Students should be kept two recitation periods and little outside work should be expected. No doubt there are objections to such a procedure. The method would seem to encourage individual effort. Some teachers would miss the opportunity to give elegant explanations to an admiring or listless class. Since the present situation is not a shining success are we not justified in trying something different in the hope of improvement?

Another suggestion is to amend the present procedure by adding a one-hour problem course to the schedule with problems on all past work as a means of keeping alive what pupils have had and of correcting deficiencies. This is done in some institutions. But it adds another hour to an already full schedule for the student.

To admit that the present regime is the best we can do is a confession unworthy a serious vocation. To attempt any serious changes

will no doubt entail difficulties and criticisms. Again it might turn out to be a case of "be damned if you do and be damned if you don't." Is it more worthy a serious profession to do and be damned or to don't and be damned?

At the present time when in many institutions the social sciences are receiving a new emphasis and taking much of the time and attention of administrative agencies, what is our tip? Do or don't? It would seem appropriate now to suggest that mathematics teachers and departments are in need of strong cooperative leadership.

Again is it possible that we have lost by the very bigness of our institutions? Would a larger number of smaller undergraduate institutions be more effective? I do not mean here merely junior colleges.

Still again it has been suggested that classes be reduced in size in subjects like mathematics. One objection to this is the comparatively heavy cost of the department. There is a tendency to value teachers by the number of student hours he can take.

In conclusion I may say the problem of effective teaching of mathematics taken in all its bearings cannot be solved by a mere wave of the hand. Not the least part of the problem is to get teachers of mathematics to adjust their old view points to new school conditions.

—W. Paul Webber

Mathematical Philosophy

Mathematics is a study which when we start from its most familiar portions, may be pursued in either of two opposite directions. The more familiar direction is constructive towards gradually increasing complexity: from integers to fractions, real numbers, complex numbers, from addition and multiplication to differentiation and integration and on to higher mathematics. The other direction which is less familiar proceeds by analyzing, to greater and greater abstractness and logical simplicity, instead of asking what can be defined and deduced from what is assumed to begin with, we ask instead what more general ideas and principles can be found in terms of which what was our starting point, can be defined or deduced. It is the fact of passing in this opposite direction that characterizes mathematical philosophy as opposed to ordinary mathematics.—Bertrand Russell in "Introduction to Mathematical Philosophy."

Notes and News

Edited by
I. MAIZLISH

On November 21, the Dr. A. M. Harding Lecture "On Other Worlds than Ours" was given at Mississippi State College under the sponsorship of the local chapter of Kappa Mu Epsilon, Mathematical Fraternity.

The three hundred eighteenth meeting of the American Mathematical Society was held at the University of California at Los Angeles, Saturday, December 1, 1934. The attendance was over fifty, twenty-seven being members of the society. The speakers included Professors E. T. Bell and A. D. Michal of the California Institute of Technology, Professor D. N. Lehmer of the University of California, and Professor E. R. Hedrick of the University of California at Los Angeles. The program was followed by a luncheon.

The departments of mathematics of Hamilton College, Syracuse University, and Colgate University met in November this year at Hamilton College. Professor Decker of Syracuse read a paper on "The Place of Rigor in Collegiate Mathematics." Professor Aude of Colgate discussed some problems from the Theory of Numbers.

A new type of computing machine, called the Trianalyst, has been invented by B. R. Wellington, of Troy, N. Y. It consists essentially of a rigid right triangle with graduated sliding scales in each of the legs; a movable hypotenuse pivoted at one end on an indicator which slides along one of the legs of the triangle; and a square with its edges perpendicular to the legs of the triangle and with its vertex sliding along the bisector of the right angle of the triangle. By means of this instrument one may add, subtract, multiply or divide numbers, raise numbers to powers or extract roots of numbers, find logarithms of numbers, find the trigonometric functions of an angle, solve quadratic, cubic, and bi-quadratic equations in one unknown, and certain simultaneous equations in two unknowns.

Professor Richard Courant, formerly director of the Mathematics Institute of the University of Gottingen, has been appointed visiting professor of mathematics at New York University.

Professor Gregory Breit, of New York University has been appointed professor of physics at the University of Wisconsin.

Assistant Professor N. H. McCoy has been promoted to an associate professorship of mathematics at Smith College.

Associate Professor Evelyn Carroll Rusk has been promoted to a professorship of mathematics at Wells College.

Assistant Professor J. H. Zant, of the Oklahoma A and M college, has been promoted to an associate professorship.

Professor W. H. Echols, professor of mathematics at the University of Virginia since 1891, died at his home in his seventy-fifth year on September 25, 1934. He was engaged in teaching until the time of his death. He was a charter member of the Mathematical Association.

Dr. Karl T. Compton, president of the Massachusetts Institute of Technology; Dr. Walter Rautenstrauch, professor of industrial engineering, Columbia University, and Dr. M. L. Crossley, chief chemist of the Calco Chemical Company, will take part on December 14 at 8:30 P. M. in a symposium on "Science in Relation to Social Growth and Economic Development," to be given under the auspices of the American Institute at the American Museum of Natural History.

Dr. Arthur H. Compton, professor of physics at the University of Chicago, was awarded the honorary degree of master of arts on the occasion of the opening of his lectures as George Eastman visiting professor at the University of Oxford.

The Marchese Marconi was elected rector by a majority of a hundred votes over the other candidate, Sir James Jeans, as the result of the St. Andrews University rectorial election.

Dr. A. E. Jolliffe, professor of mathematics in the University of London and formerly mathematical lecturer of Jesus College, University of Oxford, has been elected an honorary fellow of the college.

Mathematics section meetings as a part of the program for the Nebraska State Teachers Association were held October 25 and 26 in each of the six districts as follows: District number one at Lincoln; district number two at Omaha; district number three at Norfolk; district number four at North Platte; district number five at McCook; district number six at Chadron. The speakers included Dr. F. B. Knight, of the University of Iowa; Dr. H. R. Douglas, University of Minnesota; Dr. W. C. Brenke. University of Nebraska; Dr. A. R. Congdon, University of Nebraska; Dean William Thompson. Municipal University of Omaha; Dr. J. M. Earl, Municipal University of Omaha; Professor C. J. Frankforter, University of Nebraska.

Problem Department

Edited by
T. A. BICKERSTAFF

This department aims to provide problems of varying degrees of difficulty which will interest anyone who is engaged in the study of mathematics.

All readers, whether subscribers or not, are invited to propose problems and to solve problems here proposed.

Problems and solutions will be credited to their authors.

While it is our aim to publish problems of most interest to the readers, it is believed that regular text-book problems are, as a rule, less interesting than others. Therefore, other problems will be given preference when the space for problems is limited.

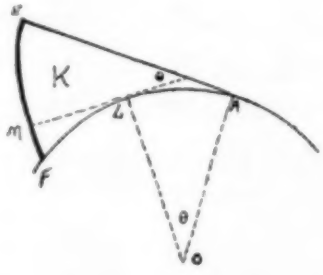
Send all communications about problems to T. A. Bickerstaff, University, Mississippi.

SOLUTIONS

No. 54. Proposed by H. T. R. Aude, Colgate University.

If a horse is tied by a rope 60 feet long, staked at the side of a vertical cylindrical tank of 100 feet diameter, find the area over which the horse can graze.

A solution has been published using the area under the involute of the circle. The following simpler solution is offered by C. E. Melville, Clark University, Worchester, Mass.



The area $K \equiv AEFA$ may be regarded as generated by the rotation of the variable radius LM as the point L moves along the circular arc FA. Accordingly,

$$dK = \frac{\overline{LM} \cdot \overline{LM}\, d\theta}{2} = \tfrac{1}{2}\, \overline{LM}^2 d\theta \text{ and } K = \tfrac{1}{2} \int_0^{6/5} \overline{LM}^2 d\theta$$

since circular arc AF = 60 ft., and AO = OL = 50 ft., θ varies from 0 to 6/5 radian.

Clearly, $LM = LF = 60 - AL = 60 - 50\theta = 10(6 - 5\theta)$

Therefore,

$$K = \tfrac{1}{2} \int_0^{6/5} 10^2 (6 - 5\theta)^2\, d\theta = -10 \int_0^{6/5} (6 - 5\theta)^2 (-5 d\theta)$$

$$= -10 \frac{(6 - 5\theta)^3}{3} \Bigg|_0^{6/5} = -10 \left[0 - \frac{6^3}{3}\right] = 720 \text{ sq. ft.}$$

Finally, the total area equals

$$1440 + \frac{\pi (60)^2}{2} = 1440 + 1800\pi.$$

No. 66. Proposed by H. T. R. Aude, Colgate University.

An ellipse with a major axis 2a and a minor axis 2b is circumscribed by a parabolic arc and a chord of the parabola perpendicular to its axis. If the axis of the parabola contains the major axis of the ellipse, show that the minimum area of the parabolic segment is 9ab/2.

Solution by Dewey C. Duncan, University of California.

The parabola whose axis lies along the x−axis has the equation:

$$Y^2 = p(X - h)$$

The ellipse whose major axis lies along the x-axis and whose semi-axes are a and b has the equation:

$$b^2X^2 + a^2Y^2 = a^2b^2$$

The condition that these be tangent is the vanishing discriminant of the Y-eliminant of their equations. This gives,

$$a^4p^2 + 4b^2(a^2ph + a^2b^2) = 0$$

Solving for p, we get, $p = \frac{2b^2}{a^2}(-h \pm \sqrt{h^2-a^2})$

Now the length of the parabolic area is h+a, taking h>0. The length of the chord of the parabola is $2\sqrt{p(-a-h)}$.

Hence the area of the parabolic segment is,

$$A = \tfrac{2}{3}(h+a) \cdot 2\sqrt{p(-a-h)}$$

Substituting the above value for p and differentiating, one obtains,

$$\frac{dA}{dh} = \frac{2\sqrt{2}b}{3a}(h+a)^{1/2}(h \pm \sqrt{h^2-a^2})^{1/2}(3 \pm (h+a)/\sqrt{h^2-a^2}]$$

The last factor alone may vanish without contradiction. This yields, h=5/4a at which the dA/dh changes from − to + indicating a minimum.

Substituting this value in the expression for A yields.

$$A = \frac{9ab}{2} \qquad \text{Q. E. D.}$$

Also solved by Karleton W. Crain, Purdue University, J. H. Neelley, Carnegie Institute of Technology, C. A. Baloff, Lincoln College (Ill.) and James A. Ward, L. S. U.

No. 67. Proposed by Walter B. Clarke.

Let A equal area of a triangle, A_0 equal area of its orthic triangle; P equal product of the sides of the given triangle, P_0 equal product of the sides of its orthic triangle. To show that $A_0.P$ equals $2A.P_0$.

Solution by C. A. Baloff, Lincoln College (Ill).

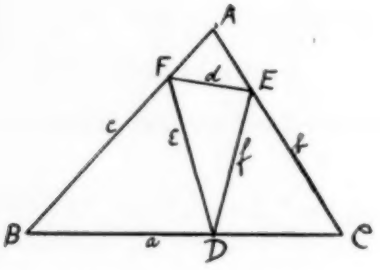
Since the area of a triangle equals the product of its sides divided by four times the radius of the circumcircle, we have $2A/A_0$ equal to $2rP/RP_0$ where r and R are the radii of the circumcircles of the orthic and given triangles respectively. But the radius of the circumcircle of a triangle is equal to twice the radius of the circumcircle of its orthic triangle. Therefore $2A/A_0$ equals P/P_0 or $A_0 \cdot P$ eaquals $2A \cdot P_0$.

Also solved by means of analytics by James A. Ward, L. S. U.

No. 68: Proposed by Walter B. Clarke.

To prove that the product of the alternate (non-adjacent) segments of the sides of a triangle as cut by altitudes equals the product of the sides of its orthic triangle.

Solution by C. A. Balof.



Let ABC be the given triangle, and DEF be its orthic triangle. Then triangles ABC, AFE, BDF, and CED are similar. (See Altshiller-Court: College Geometry, Section 136). Denote sides of triangle ABC by a, b, and c; and sides of triangle DEF by d, e, and f. Then $AE/c = d/a$, $BF/a = e/b$, $CD/b = f/c$, from whence

$AE \cdot BF \cdot CD = def$. Similarly $AF/b = d/a$, $BD/c = e/b$, $CE/a = f/c$, from whence $AF \cdot BD \cdot CE = def$.

Also solved by Richard A. Miller and James A. Ward.

No. 69. Proposed by Alexander W. Boldyreff, University of Arizona.

Prove the following theorem:

Let $R(x,y)$ be a rational function of x and y, where

$$y^n = a(x-b)^{n-1}\ (x-c)$$

a, b, c, being real constants and n a positive integer.

Then, $\int R(x,y)dx$ is integrable in terms of elementary functions.

Solved by Robert C. Yates, University of Maryland, and the proposer.

The curve $y^n = a(x-b)^{n-1}.\ (x-c)$ has a singular point at (b,o). This suggests the substitution

$$y = t(x-b)$$

yielding

$$x = (bt^n - ac)/(t^n - a)$$

$$y = t(b-c)a/(t-a)$$

$$dx = ant^{n-1} \cdot (b-c)dt/(t^n - a)$$

Thus, R(x,y)dx can be transformed quite readily into a rational function of t. Thus can the integration be effected in terms of elementary functions.

No. 71. Proposed by Alexander W. Boldyreff:

Prove that $\int_0^{\pi/4} \sec^{2n} x \, dx = \sum_{k=0}^{k=n-1} \frac{{}_{n-1}C_k}{2_k+1}$

Solved by Robert C. Yates, University of Maryland.

$$\int_0^{\pi/4} \sec^{2n} x dx = \int_0^{\pi/4} (1+\tan^2 x)^{n-1} \cdot \sec^2 x dx.$$

Setting $u = \tan x$ and $du = \sec^2 x dx$ we have

$$\int_0^1 (1+u^2)^{n-1} du = \int_0^1 [1 + {}_{n-1}C_1 u^2 + {}_{n-1}C_2 u^4 + \ldots + u^{2(n-1)}] du$$

$$= [1 + {}_{n-1}C_1 u^3/3 + {}_{n-1}C_2 u^5/5 + \ldots + \frac{u^{2n-1}}{2n-1}]_0^1$$

$$= 1 + \frac{{}_{n-1}C_1}{3} + \frac{{}_{n-1}C_2}{5} + \ldots$$

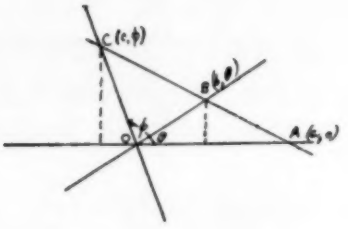
$$= \sum_{k=0}^{k=n-1} {}_{n-1}C_k/(2k+1)$$

Also solved by C. A. Balof, Lincoln College (Ill.), H. F. S. Jonah, Purdue University, West Lafayette, Indiana, and the proposer.

No. 72. Proposed by Norman Anning, University of Michigan.

In a plane, three straight lines are drawn through a point O in such a way that the whole angle around O is divided into six equal parts. A straight line not through O meets theses three lines in A, B, and C. Show that of the lengths OA, OB, OC, the reciprocal of one is equal to the sum of the reciprocals of the other two.

A solution and Generalization by A. E. Meder, Jr., New Jersey College for Women, New Brunswick, N. J.



Let the angle AOB be denoted by θ, and angle AOC by ϕ. Then the polar coordinates of A may be taken as (a,o); of B, (b,θ); and of C, (c,ϕ).

The condition that A, B, and C are collinear may be written:

$$\frac{b \sin \theta}{c \sin \phi} = \frac{a - b \cos \theta}{a - c \cos \phi}, \text{ from which we obtain simply}$$

$$b = \frac{ac \sin \phi}{a \sin \theta + c \sin (\phi - \theta)} \text{ or}$$

$$\frac{1}{b} = \frac{1}{\sin \phi}\left|\frac{\sin \theta}{c} + \frac{\sin (\phi - \theta}{a}\right|$$

a generalization of what was sought.

If $\phi = 2\theta$ we have $\quad \frac{1}{6} = \frac{1}{2 \cos \theta}\left|\frac{1}{c} + \frac{1}{a}\right|$

and if $\theta = 60°$ $\quad \frac{1}{b} = \frac{1}{c} + \frac{1}{a}$.

Also solved by Robert C. Yates, C. A. Balof, Walter B. Clarke, R. A. Miller, and the proposer.

No. 74. Proposed by H. T. R. Aude.

Prove that if the five numbers, a, b, c, d, e, are so related that $b = a + d$, $c = a + e$, and a is the geometric mean of d and e, then the sum $a^2 + b^2 + c^2$ is a square.

Solved by H. F. S. Jonah, Purdue University.

$$a^2+b^2+c^2=a^2+(a+d)^2+(a+e)^2$$
$$=3a^2+2ad+2ae+d^2+e^2$$
$$=a^2+2a^2+2ad+2ae+d^2+e^2$$
$$=a^2+2de+2ad+2ae+d^2+e^2$$

(since a $\sqrt{de}$, then $2a^2=2de$.)

$$=(a+d+e)^2$$

Therefore under these conditions

$$a^2+b^2+c^2 = (a+d+e)^2$$

that is, a square.

Also solved by R. A. Miller, C. A. Balof, Robert C. Yates, D. H. Richert, Bethel College (Kan.), Hobson M. Zerbe, Walter B. Clarke, and the proposer.

LATE SOLUTIONS

No. 65 by Norman Anning.

PROBLEMS FOR SOLUTION

No. 73. Proposed by Alexander Y. Boldyreff, University of Arizona. Prove that

$$\frac{\sqrt{20+\sqrt{20+\sqrt{20+\ldots \text{ to } \infty}}} + \sqrt{12+\sqrt{12+\sqrt{12+\ldots \text{ to } \infty}}}}{\sqrt{6+\sqrt{6+\sqrt{6+\ldots \text{ to} \infty}}}} = 3$$

Note: Due to an error in printing, this problem was stated incorrectly in the last issue.

No. 74. Proposed by Walter B. Clarke, San Jose, California.

With the notation; I for incenter, G for centroid, K for Nagel point, produce $\overline{IG}$ to V so that $2\overline{IG}=\overline{GV}$. Draw cevians through V cutting each side into two segments and show:

Half the sum of the products of the two segments of each side is equal to the difference between half the square of the sum of the sides of the triangle and the sum of the squares of the sides.

No. 75. Proposed by Alexander Y. Boldyreff, University of Arizona.

Prove that:

$$\sqrt[m]{PQ+(P^{m-1}-Q)\sqrt[m]{PQ+(P^{m-1}-Q)\sqrt{PQ+\ldots}}}\ \text{to}\ \infty$$
$$=\sqrt[n]{PQ+(P^{n-1}-Q)\sqrt[n]{PQ+\ldots}}\ \text{to}\ \infty$$

where n, m, are positive integers ≤ 2, and P, Q, are real numbers.

This problem is considered especially interesting because, among other things, it enables one to express, by means of continued radicals, any integer in an infinite number of ways. For example,

$$8=\sqrt{8+7\sqrt{8+7\sqrt{8+\ldots}}}\ \text{to}\ \infty$$
$$=\sqrt[3]{40+59\sqrt[3]{40+59\sqrt[3]{40+\ldots}}}\ \text{to}\ \infty$$

No. 76. Proposed by Walter B. Clarke.

Let A, B, and H be any two vertices and the orthocenter of a triangle. From midpoint of AB draw a line parallel to the bisector of angle AHB. Show that this line cuts the perimeter of triangle into two equal parts.

No. 77. Proposed by Richard A. Miller, University of Mississippi.

A car is traveling at the rate of 60 miles per hour along a level road when a "lug" is thrown from the rear wheel after making one-sixth revolution from the lowest point. If the diameter of the wheel is 30 inches and the lug is 6 inches from the outer circumference, determine the motion, the range, and the maximum height of the lug disregarding air resistance. Investigate the case where the road is inclined at an angle θ from the horizontal.

Technology and Mathematics

When the thinking person of the present day stops to reflect upon the facts of the wireless telegraph and the long distance telephone, not to mention many other just as important marks of human progress, and remembers in his thinking that the existence of the wireless telegraph is due to the deductions of Maxwell by means of theorems that depend upon the square root of minus one and that the possibility of the long-distance telephone depended upon investigations of Pepin by means of theorems that depend more directly upon the modern theory of expansions in fundamental functions, he appreciates to the full the power of this branch of learning. —James Byrnie Shaw in "The Philosophy of Mathematics."

Book Reviews

Edited by
P. K. SMITH

Solid Geometry. By Elizabeth B. Cowley. Silver, Burdett and Company, New York, 1934—IX+230 pages.

One of the features of this text, written primarily for use in the high schools, is its flexibility as to the type of course desired and as to individual differences among students. The subject matter covered in the text is essentially that usually covered in Solid Geometry texts.

At the ends of the chapters there are a summary of the theorems covered, a set of review exercises, a set of optional review exercises, a matching test, and a multiple choice test.

In the preface six plans of presenting the course are outlined. The first three plans concern Solid Geometry exclusively, while the last three outlines refer to a combination of Plane and Solid Geometry. The combination plans are to be covered in two semesters. I shall give the first three plans only.

Plan A calls for a review of the commonly-used theorems of Plane Geometry, a study of the introduction to the text, a comprehension of the meaning of the eighty-three theorems in the text, ability to prove fourteen marked theorems, a study of the review exercises immediately following the marked theorems, and the matching test—obviously a brief course. Plan B includes Plan A, selections from the review exercises that follow each group of theorems, the summary, and the multiple choice test. Plan C includes Plan B, selections from the optional review exercises, ability to prove starred theorems from the College Entrance Examination Board, and the Trigonometric functions.

For an exceptional group of students an instructor might desire to cover all theorems. The author failed to suggest this case in any of her first three plans.

An interesting feature of the text is its splendid illustrations. There is a picture in book VIII of the metal gondola of a stratosphere balloon. Many diagrams from Engineering for calculations on steel fabrication are given. There is, indeed, a splendid selection of pictures. A picture of Hadrian's Bridge and Mausoleum, a picture of the pyramidal temples of the Mayan peoples, and a picture of a Kansas Tornado are given. These illustrations should surely aid in creating interest in the subject.

The diagrams are well done, with shading used on many of them. The mechanical make-up is excellent, and the size is 6"×8".

After examining a text so elaborately compiled, the writer is inclined to feel that some of our present day writers of elementary text books put too much stress on arrangement of material and plans of study. Some one has said, "It is fine to plan your work if afterward you work your plan."—P. K. SMITH.

Differential and Integral Calculus. By Clyde E. Love, MacMillan Company, New York, March, 1934—xv+383 pages.

The review to follow concerns the third edition of this very popular and widely adopted text. The third edition varies in no fundamental manner from the second. In the preface it is noted that the "only note-worthy addition is a brief treatment of approximate solutions of equations." Some of the discussions of theory have been amplified, and in some sets the problems have been varied and increased.

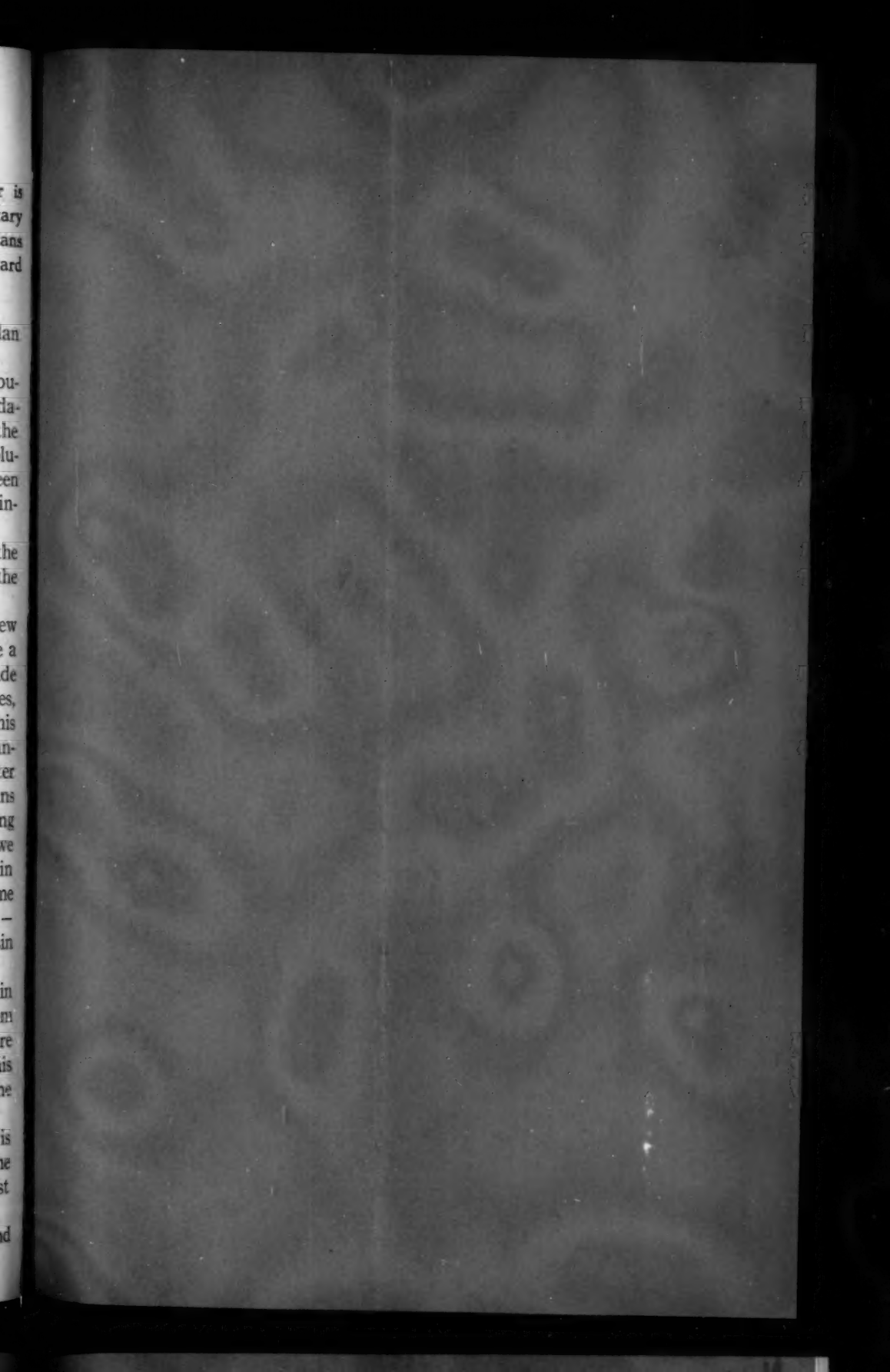
The author calls attention to the fact that the chapters on the indeterminate forms and curve tracing have been postponed and the integral calculus thereby advanced.

There are a few changes which the writer would make in the new edition. On page 20 the following statement is found: "Let P′ be a fixed point of a plane curve and P′ a neighboring point. If P′ be made to approach P along the curve, the secant P P′ evidently approaches, in general, a definite limiting position (PT in the figure) and in this position, at the instant when P′ coincides w th P, it is called the tangent to the curve at P, or is said to touch the curve at P". The writer is unable to see the necessity or desirability of stating that P′ attains its limit P. Why not merely state that the tangent is the limiting position of the secant and leave the question with that? When we give the geometric meaning to the derivate the student will be in difficulty when P′ actually reaches P, since the $\Delta y/\Delta x$ will then become o/o. A minor change would have been to use the formula $\sin A - \sin B = 2 \cos \frac{1}{2}(A+B) \sin \frac{1}{2}(A-B)$ in deriving the fomula for d/dx sin x on page 63.

The writer questions the use of the differential on the figure in visualizing the typical element while applying the fundamental theorem for definite integrals. Why not use Δx_i instead of dx on the figure and sum the elements $\alpha(x_i)\,\Delta x_i$. The use of the differential in this situation, while convenient, leads to loose thinking in the use of the above theorem.

In spite of the above criticisms, the writer feels that this text is one of the best in the field. The chapters on the applications of the calculus to theoretical mechanics are superior to those found in most calculus texts.

The size of the text is the same as that of the second edition and the color is grey.---P. K. Smith.



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